Eur. Phys. J. B **18**, 459–466 (2000)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Cellular model for the compaction of a vertically tapped granular column

J.-M. Hertzsch[a]

Brunel University, Department of Mathematical Sciences, Uxbridge, UB8 3PH, UK

Received 3 March 2000 and Received in final form 21 July 2000

Abstract. A cellular model for the compaction of granular material is described. It takes into account horizontal redistribution as well as vertical transfer of particles. Parameters are the width of the horizontal redistribution and the settling probability. Numerical simulations of the behaviour of a granular column in a container are shown as an example, and the evolution of some characteristic features over time has been followed for some typical configurations. Experimental results for the time evolution of the density can be reproduced for a settling probability proportional to the unoccupied spaces for particles in the lower cells.

PACS. 45.70.-n Granular systems

1 Introduction

The ubiquitous presence of granular materials in nature, their technological importance and their very interesting physical behaviour lead to considerable scientific interest in such systems. One question of particular technological interest is the compaction of granular materials in silos, transport containers, and other storing facilities, or during manufacturing processes [1]. Everyday experience shows that excitations in the form of tapping, shaking or of vibrations have a pronounced effect on this phenomenon.

In the present paper a simple numerical model is presented for the redistribution and compaction of granular material in a container as a consequence of a succession of discrete vertical excitations (for example, strokes to the bottom of the container). The results will be compared with recent experimental findings [2–4] which have been theoretically explained in various ways, using a free-volume concept [5], Monte-Carlo simulations of frustrated lattice gases [6] or making use of results for supercooled liquids [7].

The time evolution of the model presented here is such that the actual states of its constituent parts depend on the states at the previous time step in a determined way. It is convenient to describe this evolution in terms of a cellular model [8]. In particular, the time dependence of the compaction will be discussed for different rules governing the settling of the powder. It will be assumed that the material comes to rest after each tapping stroke (as opposed to vibratory compacting at higher frequency described in [9]).

[a] *Present address:* Institut für Mineralogie, Museum für Naturkunde, 10099 Berlin, Germany.
e-mail: jan-martin.hertzsch@rz.hu-berlin.de

2 Cellular model

2.1 General

If $N_\alpha(t)$ denotes the state of the cell with the general index α after a certain time step t, the state after the next time step is determined by a "transition matrix" $\mathcal{A}$ which represents the "rules of the game" and by the states of all cells with indices β at the same time step t which by virtue of non-zero elements of $\mathcal{A}$ determine have an influence on cell α:

$$N_\alpha(t+1) = \sum_\beta \mathcal{A}_{\alpha\beta}(t) N_\beta(t). \qquad (1)$$

In a more general approach, the values of the elements $\mathcal{A}_{\alpha\beta}$ may depend on the actual states of the cells.

Granular systems, considered at the level of individual grains, make no exception. Indeed, they have been modelled in such form quite successfully. Description in form of a cellular automaton is also possible for systems where one abstracts from the single-particle level and literally divides the granular system in a number of cells which can be occupied by a varying number (occupation number) of grains, the latter characterising the actual state of each particular cell. The "transition matrix" $\mathcal{A}_{\alpha\beta}$ is in this case described by the probabilities for particles being transferred from cell β to cell α which may depend on the momentary occupation number of the cells, and it may be convenient to describe the evolution between two time steps as a succession of two or more intermediate steps, thereby defining $\mathcal{A}_{\alpha\beta}$ by the combined effect of the latter:

$$N_\alpha(t+1) = \sum_\gamma \mathcal{A}_{2,\alpha\gamma} \left(\sum_\beta \mathcal{A}_{1,\alpha\beta} N_\beta(t) \right). \qquad (2)$$

The abstraction may even be led as far as the occupation number is considered simply the value of the mass of granular material in each of the cells.

2.2 Redistribution of particles in a column

For the purpose of this model, the volume of a container filled with a granular material shall be divided in cells with rectangular borders. For simplicity, a two-dimensional system will be considered first. Let x and z denote the horizontal and vertical coordinates, respectively. The walls of the container shall be parallel to the z-direction at $x = w_1$ and $x = w_2$ ($w_1 < w_2$) and extend high enough that no particles can be spilled. Gravity acts in negative z-direction. A cell shall be denoted by the coordinates of its lower left corner (x, z). The number of particles in cell (x, z) at time step t is $N(x, z; t)$. Each cell can contain up to a maximum number $N_{\max}$ of particles, but may not necessarily be filled up to capacity.

If the system is tapped from below, a certain number of particles will be lifted from their original position as described above, redistribute over their original and the neighbouring cells and settle again. Each vertical excitation (tap) shall correspond to a step in time. Between successive taps, the particles are allowed to come to rest. The new number of particles in one cell is calculated according to

$$N(x,z;t+1) = N(x,z;t) - \sum_{\xi,\zeta} N(x,z \to \xi,\zeta) + \sum_{\xi,\zeta} N(\xi,\zeta \to x,z) \quad (3)$$

where the second term on the right side corresponds to the number N_{out} of particles leaving this cell and the third one stands for the number N_{in} of particles being transferred into it.

The following basic types of transfer from cell (x, z) will occur at each time step t: "spreading" if $N(x \pm 1, z) < N_{\max}$ (which may lead to particles "piling up" in some cells) and "compaction" if $N(x + \delta, z - 1) < N_{\max}$ ($\delta \in \{-1, 0, +1\}$). It will be assumed that the vertical redistribution occurs after the horizontal one.

2.3 Horizontal spreading

One may expect that the number of particles which are transferred horizontally from one cell to another will be proportional to the number of particles occupying the first one:

$$N(x,z \to \xi,z) = P(x,z \to \xi,z) N(x,z;t). \quad (4)$$

Now the probability P will be estimated with which the horizontal redistribution occurs in order to calculate the number $N(x, z \to \xi, z)$ of particles transferred from one given cell (x, z) to another one (ξ, z).

$$P(x,z \to \xi,\zeta) = P_{\text{h}} P_{\text{v}}. \quad (5)$$

The horizontal redistribution of the particles from box (x, z) to box (ξ, z) due to "spreading" over a certain length is assumed to take place with a probability G according to a symmetric binomial distribution with the general form

$$G(m;n) = \binom{n}{m} 2^{-n} \quad (6)$$

($0 \le m \le n$, $m, n \in N$) where $n + 1$ is the width of the distribution, and m the variable. Let in our system M denote the maximal number of cells in either direction over which horizontal redistribution is assumed to occur, and the distribution shall be centered on the box (x, z) from which the grains spread out. Then the width of the distribution equals $n = 2M+1$, and $m = (\xi - x) + M$ with $x - M \le \xi \le x + M$. Therefore the probability distribution is given by

$$G(\xi - x; 2M) = G_M(\xi - x) = \binom{2M}{\xi - x + M} 2^{-2M}. \quad (7)$$

M may serve as a measure for the excitation strength – the higher the latter is, the more the grains are likely to spread over a larger area. The establishment of a more detailed relationship will be the subject of forthcoming investigations.

Since the motion of particles in lower layers is restricted by the presence of the higher layers, M must depend on the height z. This is supported by a recent analysis of the momentum propagation through a one-dimensional array of solid particles [10,11], where for impacts at one end of the column the momentum of particles moving off the other end was calculated. The height h_{lift} to which a layer of particles is lifted will depend on the fraction of momentum p_{layer} (transferred by each stroke to the column) which is finally transferred to it as $h_{\text{lift}} \propto \sqrt{p}_{\text{layer}}$. However, for an exact analysis of a three-dimensional multilayered array of granular material this analysis would need considerable extension which is outside the scope of the present article.

For the purpose of demonstration of the cellular redistribution model, the dependence $M(z)$ is therefore chosen as follows (see also Fig. 1):

$$M(z) = \begin{cases} \dfrac{M_{\max}}{H - H_0}(z - H_0 + 1) & \text{if } H_0 \le z \le H \\ 1 & \text{else} \end{cases} \quad (8)$$

where spreading farther than into the immediately adjacent cells is only possible above a certain height H_0, and the width of the distribution increases linearly towards the uppermost layer at $z = H$.

2.4 Wall effects

For horizontal redistribution in a finite container, collisions of the grains with the walls have to be included. The degree of inelasticity of those collisions can be expressed

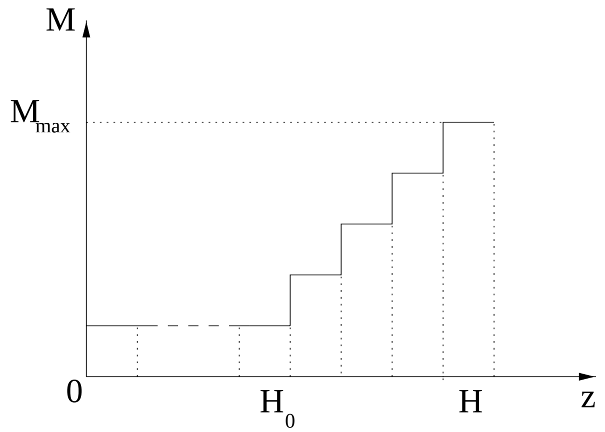

Fig. 1. Dependence of the horizontal distribution width M on height z (example: $M = 5$).

by a factor $\beta \in [0, 1]$ (the restitution coefficient) which is the ratio of the velocity of a particle after its collision with the wall to the velocity before. This factor can be related to the material properties for the given combination of wall and granular particles, namely the restitution coefficient which may, in turn, depend on the impact velocity [12]. Elastic rebound corresponds to $\beta = 1$.

The new horizontal position ξ of a particle is the result of a shift $\Delta x \in \mathbf{R}$ from the original position x, where Δx is determined by the horizontal transfer probability P_h. It is $\xi = x + \Delta x$ if no collision with the wall takes place. If this displacement would lead to a position ξ^* outside the container, that means either in cell w_1 or further to the left, or in cell $w_2 + 1$ or further to the right, the particle is assumed to be reflected (in case of elastic interaction) from the wall by the difference between ξ^* and the x-coordinate of the box just left of the wall. In the case of inelastic interaction with the wall, this distance is diminished by the restitution coefficient β, since the reduced velocity implies a correspondingly shorter travel distance per time step (all other interactions of the grain during its flight shall be neglected).

If $\xi^* < w_1$, the new position is $\xi = w_1 + \beta (w_1 - 1 - \xi^*)$, and if $\xi^* > w_2$, it will be $\xi = w_2 - \beta (\xi^* - w_2 + 1) = w_2 + \beta (w_2 - 1 - \xi^*)$ so that we can write:

$$\xi = \begin{cases} w_i + \beta (w_i - \xi^*) & \text{if } \xi^* < w_1 \text{ or } x + \Delta x > w_2 \\ & (i \in \{1, 2\}) \\ x + \Delta x & \text{else} \end{cases} \tag{9}$$

assuming that the maximal distance covered by a horizontally transferred particle does not exceed the width of the container, *i.e.* $M < w_2 - w_1$.

The probability distribution will therefore be affected by collisions of particles with the wall in such a way, that the number of particles which would be transferred to a cell $\xi^* < w_1$ or $\xi^* > w_2$ will be added to cell ξ, too. From (9) follows

$$\xi^* = \frac{w_i - \xi}{\beta} + w_i - 1. \tag{10}$$

The total probability of horizontal transfer from cell x to cell ξ is therefore given by the modified distribution

$$G_\mathrm{total}(\xi - x) = G_M(\xi - x) + \sum_{i=1}^{2} G_M\left(\left(\frac{w_i - \xi}{\beta} + w_i - 1\right) - x\right). \tag{11}$$

Combining the above formulae, the horizontal transfer probability can be formulated as

$$P_\mathrm{h}(x, z \to \xi, \zeta) = G_\mathrm{total}(\xi - x), \tag{12}$$

and the number of particles transferred horizontally is

$$N(x, z \to \xi, z) = G_\mathrm{total}(\xi - x)\, N(x, z, t). \tag{13}$$

Horizontal gain and loss term for cell (x, z) are therefore expressed by

$$N_\mathrm{in,h}(x, z; t) = \sum_{\xi = w_1}^{w_2} G_\mathrm{total}(\xi - x)\, N(x, z, t) \tag{14}$$

$$N_\mathrm{out,h}(x, z; t) = \sum_{\xi = w_1}^{w_2} G_\mathrm{total}(x - \xi)\, N(\xi, z, t). \tag{15}$$

2.5 Vertical particle transfer

Since no cell is allowed to contain more than N_max particles at once, it will be assumed that any excess particles which are transferred to a certain position ξ during the horizontal redistribution will be deposited in the cell above the current one. The number of particles in cell (x, z) after horizontal transfer is therefore given by

$$N_\mathrm{h}(x, z; t) = \max\left(\left(N(x, z; t) + N_\mathrm{in,h}(x, z; t) - N_\mathrm{out,h}(x, z; t)\right), N_\mathrm{max}\right), \tag{16}$$

and the excess number of particles which must stay in the upper cell is

$$N_\mathrm{excess}(x, z; t + 1) = \max\left(\left(N(x, z; t) + N_\mathrm{in,h}(x, z; t) - N_\mathrm{out,h}(x, z; t) - N_\mathrm{max}\right), 0\right). \tag{17}$$

It is necessary to evaluate the layers of the granular column which are affected by the excitation from bottom to top in order to take into account this transfer in the correct way. The number $N_\mathrm{excess,h}(x, z, t + 1)$ contributes to the gain term $N_\mathrm{in}(x, z + 1, t)$ for cell $(x, z + 1)$ in the time step of the current excitation. This way, possible vertical "piling" of the granular material is accounted for.

Vertical compaction (as opposed to the upward transfer just discussed) is assumed to take place after the horizontal spreading (and "piling" in case it takes place) of the material. It means that particles will fall from a cell at height z to another one at height $z-1$ if the lower cell can still accommodate particles, *i.e.* if $N(x, z-1) < N_{\max}$. On the other hand, no more than the actual number of particles in the upper cell $N(x, z)$ can leave it. With

$$N_{\mathrm{h+e}}(x, z; t) = N_{\mathrm{h}}(x, z; t) + N_{\mathrm{excess}}(x, z-1; t) \quad (18)$$

is the number of particles after horizontal transfer and "piling" have taken place, the maximum number of particles transferred vertically to a lower cell will therefore be $N_{\mathrm{v,max}}(x, z \to x, z-1) =$

$$\begin{cases} \min(N(x, z; t), N_{\max} - N_{\mathrm{h+e}}(x, z-1; t)) & \text{if } z > 1 \\ 0 & \text{if } z = 1. \end{cases} \quad (19)$$

Of course, no particles are allowed to pass through the bottom of the container.

The available space in a lower cell will not necessarily be fully occupied by settling particles. To account for this, $N_{\mathrm{v,max}}(x, z \to x, z-1)$ is multiplied by a number $R(x, z) \in [0, 1]$ which stands for the settling probability. It will differ from cell to cell, influenced by the local density and other effects like friction with the walls of the container. While the latter can be accommodated by a certain factor, the settling will be the less probable the more material already is already in the lower cell. A possible assumption taking account of the diminishing probability of finding empty spaces in the lower cell with increasing occupation of the same is

$$R(x, z) = \frac{N_{\max} - N(x, z-1)}{N_{\max}} \quad (20)$$

which yields $R(x, z) = 1$ for $N(x, z-1) = 0$ and $R(x, z) = 0$ for $N(x, z-1) = N_{\max}$. This will – as shown below – lead to a realistic behaviour of the system. In contrast to this, a settling probability independent on the occupation of the lower cell, *i.e.* a settling factor with $\frac{\partial R(x,z)}{\partial N(x,z-1)} = 0$ can not account for the observed behaviour of vertically excited granular columns.

Thus, the final number of particles in a cell after excitation becomes:

$$\begin{aligned} N(x, z; t+1) = N_{\mathrm{h+e}}(x, z; t) \\ + R(x, z+1) N_{\mathrm{v,max}}(x, z+1 \to x, z) \\ - R(x, z) N_{\mathrm{v,max}}(x, z \to x, z-1) \end{aligned} \quad (21)$$

where equations (13, 18) supply the numbers of particles transferred due to horizontal spreading, and equation (19) those due to vertical settling.

3 Numerical simulations

The granular column is subdivided in cells which are initially filled with a random number of particles between 0 and $N_{\max}$. Each excitation is assumed to initiate redistribution of the particles and corresponds to one time step. Between the excitations, the granular material shall come to rest. The occupation number of the cells is calculated as described in the previous section.

Simulations have been performed for a system consisting of an array of 50×50 cells. Impacts with the container walls are assumed to be elastic ($\beta = 1$). The maximal number (or mass) of particles per cell is 50, and the maximal width of the probability distribution for horizontal spreading of the particles was chosen to be $2M + 1 = 11$. Different initial configurations and settling factors have been used, and in order to simulate different degrees of roughness of the container walls, the settling factor for the cells next to them was chosen to be either the same as in the interior of the column (resp. underlying the same conditions as there) or to differ from it by a certain factor. The strength of the excitation is always the same.

Simulations of the following initial situations are presented:

- uniform initial distribution of grains, uniform settling behaviour (Fig. 2);
- uniform initial distribution of grains, settling in cells next to the walls slower than in interior of container by a factor 0.5 (Fig. 3);
- random initial distribution of grains, uniform settling behaviour (Fig. 4);
- random initial distribution of grains, settling in cells next to the walls slower than in interior of container by a factor 0.5 (Fig. 5).

In order to demonstrate the consequences of other compaction rules, Figure 6 shows a simulation of a system with a constant settling factor $R(x, z) = 0.5$.

These simulations can be viewed in detail as animated images *via* the WWW at `http://www.brunel.ac.uk/~masrjmh/Animations/` where also some results for other combinations of initial conditions are shown.

4 Results

Clearly visible in all simulations is the consolidation of the material in the lower cells and the gradual depletion of the upper part of the column. The horizontal particle distribution, if not homogeneous, is also found to be levelled with advancing compaction. The relative strength of these effects depends on settling factor and spreading width.

The main attention shall be given to the evolution of the density of the granular column. To this end, the total height H of the column has been plotted against the number of time steps (equal to the number of taps) in Figure 7. For the model using equation (20), a dependence

$$H(T) = H(\infty) + (H(0) - H(\infty)) \exp\left(\frac{\tau_1 - T}{\tau_2}\right) \quad (22)$$

with constants τ_1 and τ_2 is found while the use of a constant settling factor leads to a linear decay:

$$H(T) = H(0) - (H(0) - H(\infty)) \frac{T - \theta_1}{\theta_2} \quad (23)$$

with constants θ_1 and θ_2.

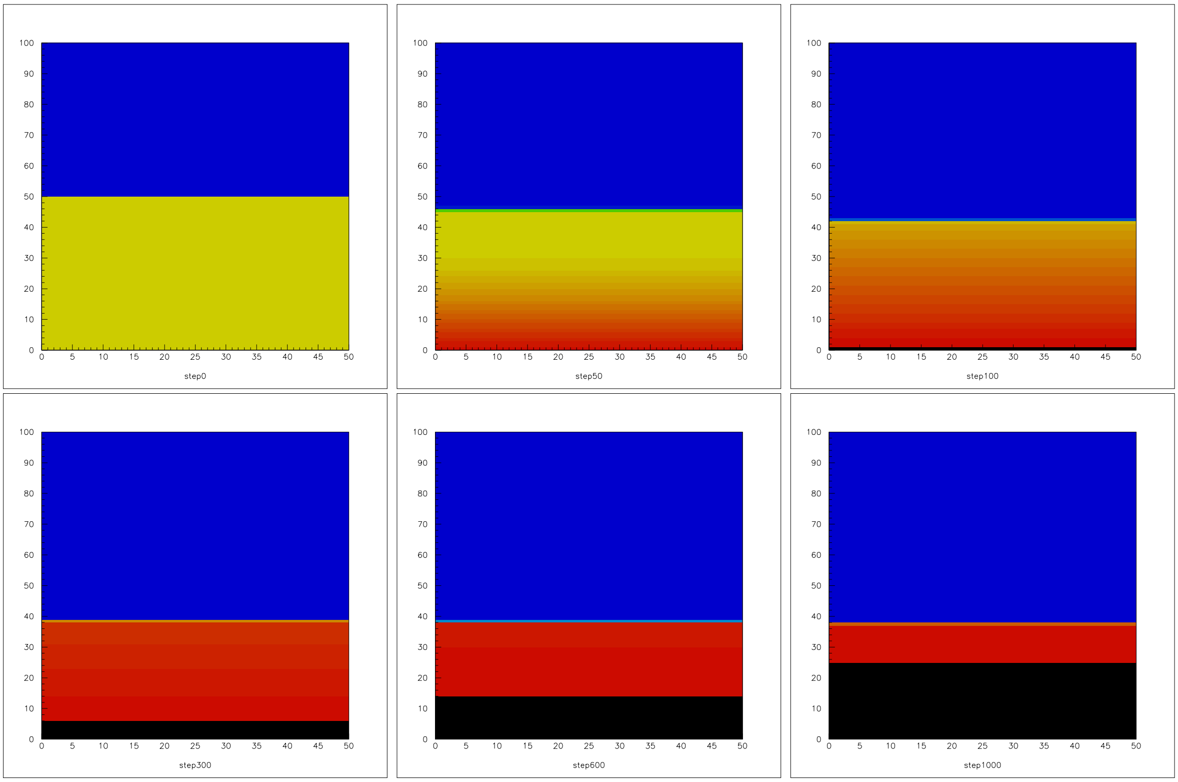

Fig. 2. Redistribution of particle density in a box. Initial distribution: uniform with $n(x, y; 0) = 0.75 n_{\max}$; uniform settling behaviour. Top from left to right: situation before the first, after the 50th, and after the 100th excitation. Bottom from left to right: situation before the 300th, 600th, and 1000th excitation.

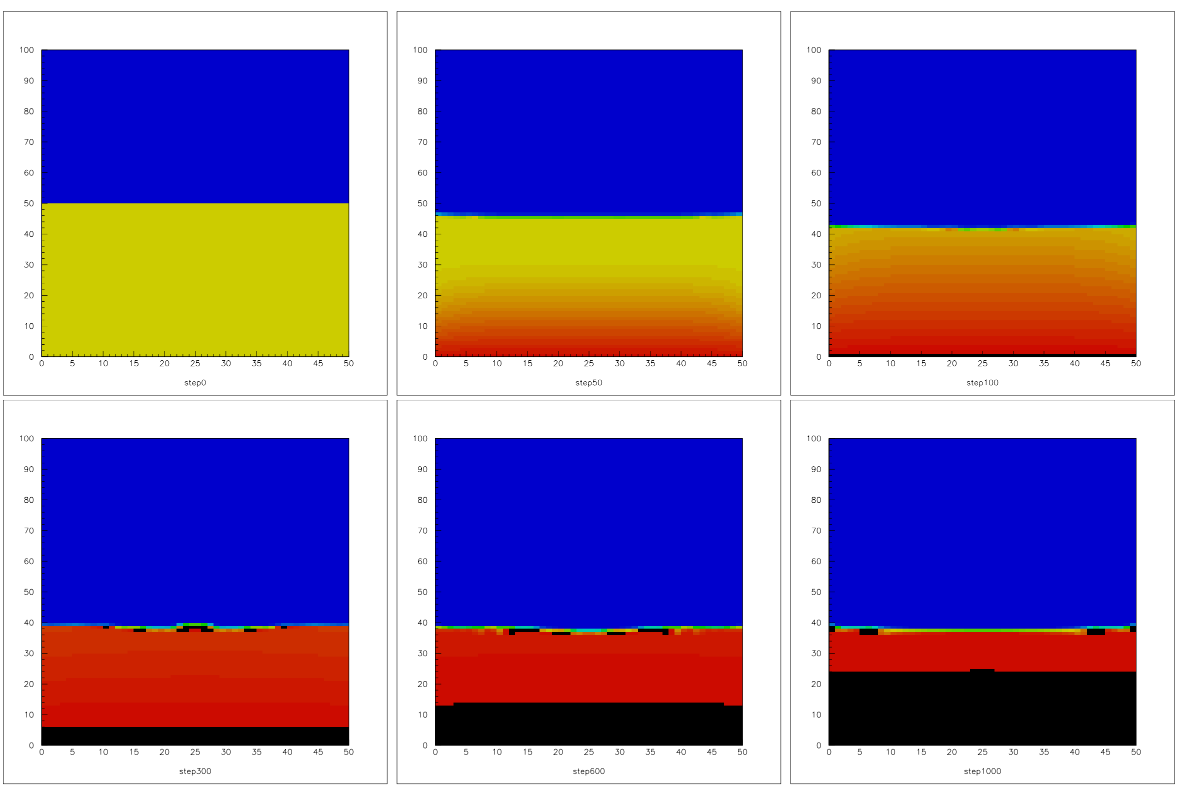

Fig. 3. The same as in the previous figure, but settling at walls slower than in interior of container. Note the regions of enhanced density in the central part of the upper layers which indicates arching.

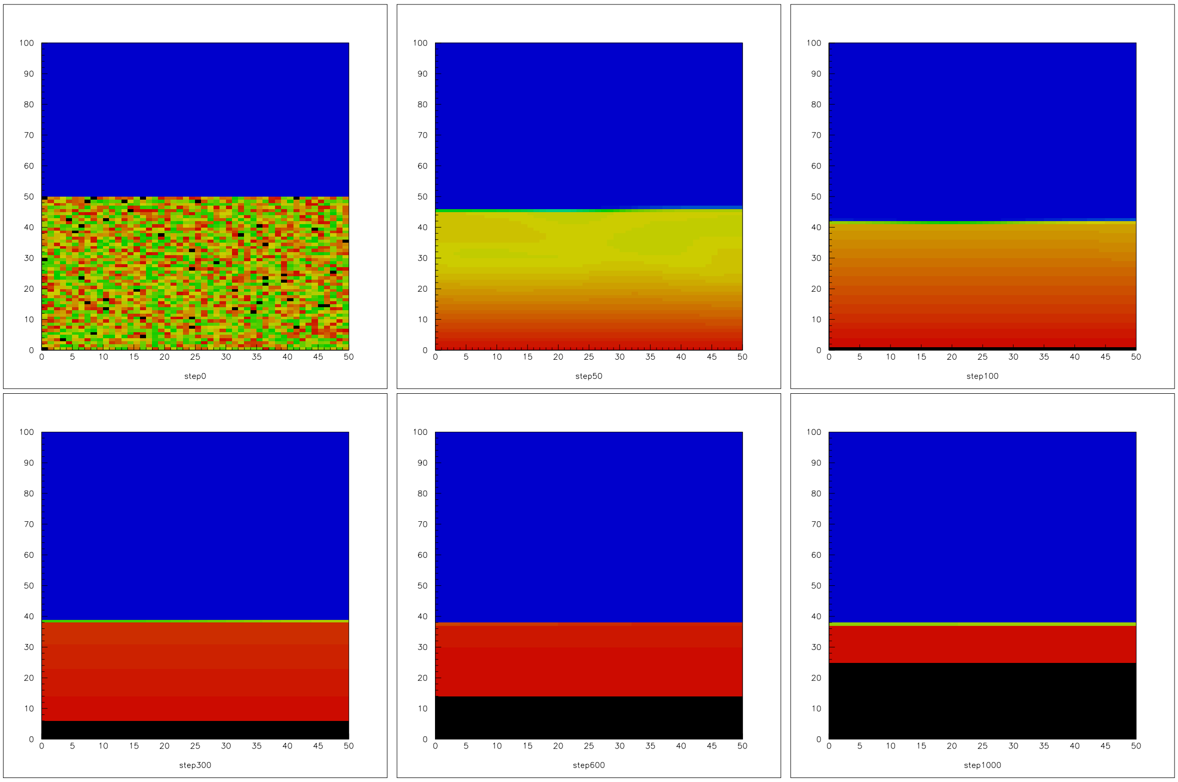

Fig. 4. The same as in Figure 2, but for a random initial distribution with $0.5n_{\max} \leq n(x, y; 0) \leq n_{\max}$; settling behaviour at walls identical to settling behaviour in bulk.

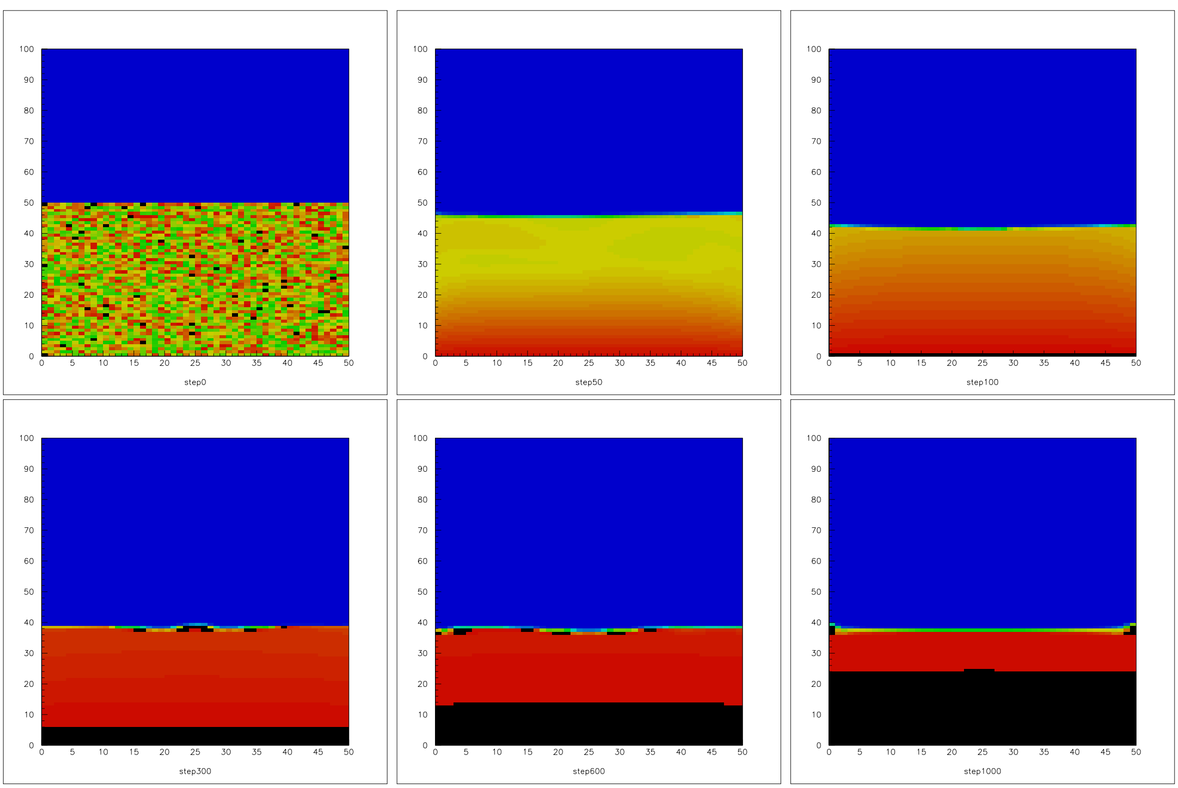

Fig. 5. The same as in the previous figure, but settling at walls slower than in interior of container. In some stages of compaction, the density in central parts of the upper layers is enhanced compared to the situation with uniform settling.

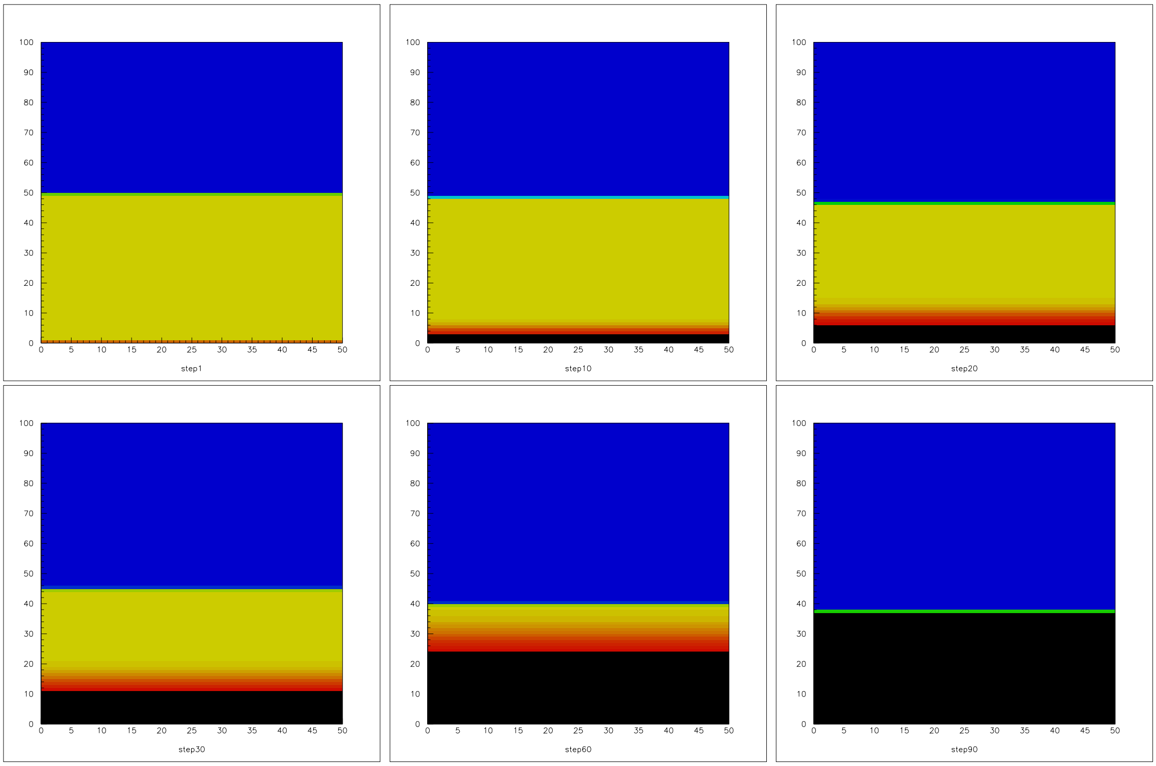

Fig. 6. Redistribution of particle density in a box. Initial distribution: uniform with $n(x, y; 0) = 0.75n_{\max}$; constant settling factor $R(x, z) = 0.5$; uniform settling behaviour. Top from left to right: situation after the first, 10th, and 20th excitation. Bottom from left to right: situation after the 30th, 60th, and 90th excitation.

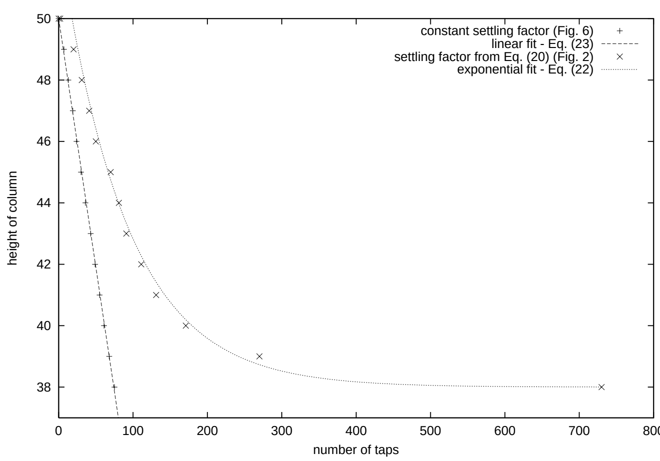

Fig. 7. Height H of granular column *vs.* number of time steps T.

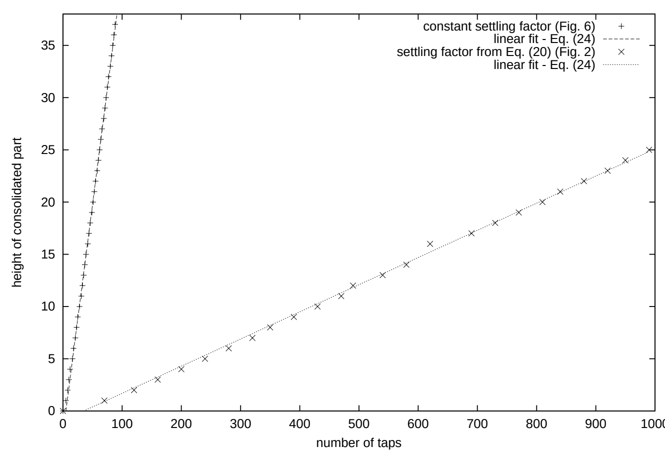

Fig. 8. Height H_c of consolidated layer *vs.* number of time steps T.

Also, in order to illustrate the progressive consolidation of the granular matter, the height H_c of that part of the column in which the density has reached its maximum has been estimated from the simulation data after each step and plotted in Figure 8. Interestingly, this grows in a linear progression no matter which settling law is chosen:

$$H_c(T) = H_c(\infty)\frac{T - \Theta_1}{\Theta_2} \tag{24}$$

with constants Θ_1 and Θ_2. The apparent discrepancy that $H(\infty) = H_c(\infty) + 1$ instead of being equal to each other is due to the subdivision of the column in discrete layers which need not be filled completely.

In order to compare the result of the simulation using the settling law from equation (20) with experiments [2,3], the overall density $\rho(T)$ (T being the number of taps) has been calculated, and the quantity

$$f(T) = \frac{\rho(\infty) - \rho(0)}{\rho(\infty) - \rho(T)} - 1 \tag{25}$$

has been plotted over T in Figure 9. While it follows a power law $f(T) \propto T^k$ with $k > 1$ in the earlier stages of compaction, in its advanced stages it can indeed be fitted

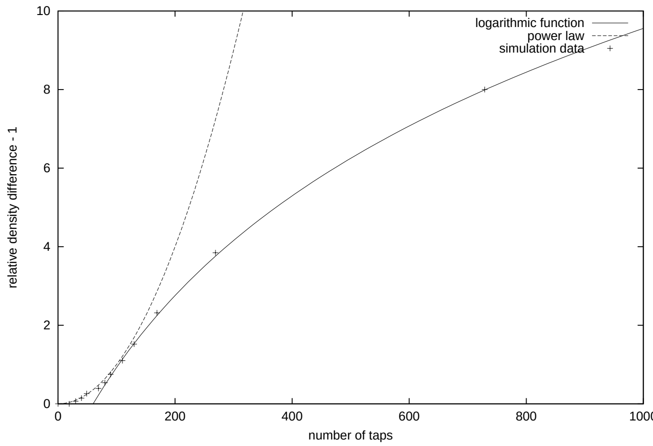

Fig. 9. Relative density difference according to equation (25) *vs.* number of time steps T for model shown in Figure 2.

by a function of the form

$$f(T) = a \ln\left(1 + \frac{T - T_*}{b}\right) \quad (26)$$

as proposed in [2] and derived in [5].

In contrast to this result, the model with a settling factor independent of the occupation of the lower cells (as shown in Fig. 6) is characterised by an a linear decay of the column height with time. Also, the height of the consolidated part of the column increases in a linear manner:

$$\frac{\mathrm{d}H}{\mathrm{d}T} = \alpha, \; \frac{\mathrm{d}H_\mathrm{c}}{\mathrm{d}T} = \beta \quad (27)$$

with α, β being constants which depend on the parameters of the simulation. The density depends on time or number of steps T in the following way:

$$\rho(T) = \rho(0)\frac{1}{H(0) - CT} \quad (28)$$

until the maximum is reached. $C = \frac{\alpha}{H(0)}$ is a constant. This result does not agree with experimental results, so that the model using equation (20) for the settling is to be preferred.

Remarkable is the effect of "rough" walls where particles can not settle as easily as in the interior of the container as shown in Figures 3 and 5. It is not only noticed in the cells immediately next to the walls, but the slowdown of the settling propagates into the next columns inward, and moreover, some areas of higher density appear above less compacted parts of the column. This has not been included in the model *a priori*. One could compare this effect with so-called "arching" as described in [13–15]. This needs to be studied in more detail.

5 Conclusion

In its present state, the model is based on data which need to be estimated experimentally. Those are the restitution coefficient between wall and particles and the settling factor. The latter is expected to depend on the shape and size of the particles as well as on the surface interactions between them properties of the particles. Also, the relationship between excitation strength (*i.e.* the momentum transferred to the granular column in each stroke) and the horizontal redistribution probability needs to be established.

The outcome of the model depends significantly on the compaction law. It has been found that the settling probability must depend on the number of available spaces in lower cells, otherwise it is impossible to reproduce experimental results. If the vertical downward motion of grains follows equation (20), it is possible to draw a realistic-looking picture of the compaction processes. While the time evolution of the height of the column can be described by an exponential law and the height of its maximally compacted (consolidated) part increases as a linear function of the number of taps, the density is found to depend on time in a logarithmic fashion in agreement with findings in [2,3,5,6].

The constants in the equations governing the time dependence of the main features still need to be related to the initial conditions of the simulations. This as well as the observed "arching" for high friction between granulate and walls should be subject of further investigation.

This work has been supported by EPSRC.

References

1. *Vibratory Compacting, Principles and Methods*, edited by H.H. Hausner, K.R. Roll, P.K. Johnson, Vol. 2 of *Perspectives of Powder Metallurgy* (Plenum Press, New York, 1967).
2. J.B. Knight, C.G. Fandrich, C.N. Lau, H.M. Jaeger, S.R. Nagel, Phys. Rev. E **51**, 3957 (1995).
3. E.R. Nowak, M. Povinelli, H.M. Jaeger, S.R. Nagel, J.B. Knight, E. Ben-Naim, *Studies of granular compaction*, in *Powders & Grains 97*, edited by Behringer, Jenkins (Balkema, Rotterdam, 1997), pp. 377–380.
4. E.R. Nowak, J.B. Knight, E. Ben-Naim, H.M. Jaeger, S.R. Nagel, Phys. Rev. E **57**, 1971 (1998).
5. T. Boutreux, P.G. de Gennes, Physica A **244**, 59 (1997).
6. M. Nicodemi, A. Coniglio, H.J. Herrmann, Phys. Rev. E **55**, 3962 (1997).
7. D.A. Head, Phys. Rev. E **62**, 2439 (2000).
8. S. Wolfram, *Cellular Automata and Complexity* (Addison-Wesley, Reading, Mass., Wokingham, 1994).
9. C. Josserand, A. Tkachenko, D.M. Mueth, H.M. Jaeger, Phys. Rev. Lett. **85**, 3632 (2000).
10. V.F. Nesterenko, J. Appl. Mech. Tech. Phys. (USSR) **5**, 733 (1983).
11. E.J. Hinch, S. Saint-Jean, Proc. R. Soc. Lond. A **455**, 3201 (1999).
12. W. Goldsmith, *Impact. The theory and physical behaviour of colliding solids* (Edward Arnold, London, 1960).
13. J.P. Wittmer, M.E. Cates, P. Claudin, J. Bouchaud, *Arching effects in the stress distribution of sandpiles*, in *Powders & Grains 97*, edited by R.P. Behringer, J. Jenkins (Balkema , Rotterdam, 1997), pp. 303–306.
14. J.P. Wittmer, M.E. Cates, P. Claudin, J. Phys. France I **7**, 39 (1997).
15. J. Duran, E. Kolb, L. Vanel, Phys. Rev. E **58**, 805 (1998).